**Mathematical modelling**

**Homework**

**Task 1. Introduction. Mathematical models of flight dynamics**

**Methodical instructions**

In Lecture 1, general principles of mathematical modelling were discussed. The structure of mathematical models was described. A classification of mathematical models was given. As an example, we considered the problem of a body falling under its own weight.

The subject of Task 1 is mathematical models of flight dynamics, which are generalizations of the mathematical model of the body falling process. A description of these models is given in the Appendix of Lecture 1. In the following tasks, it is required to indicate the marked characteristics of the corresponding mathematical model, just as it was done in lecture for the model of the body falling.

**Variants**

1. For the mathematical model of the probe movement, determine the object of study, coordinate system, output parameters.
2. For the mathematical model of the probe movement, determine the state functions, causal relationships, input parameters.
3. For the mathematical model of the probe movement, determine the reason for the evolution, the conditions of the model applicability, the independent variables.
4. For the mathematical model of the probe movement, determine the cause of evolution, the object of research, the conditions of the model applicability.
5. For the mathematical model of the probe movement, determine the input parameters, causal relationships, state functions.
6. For the mathematical model of the missile flight, determine the object of study, coordinate system, output parameters.
7. For the mathematical model of the missile flight, determine the state functions, causal relationships, input parameters.
8. For the mathematical model of the missile flight, determine the reason for the evolution, the conditions of the model applicability, the independent variables.
9. For the mathematical model of the missile flight, determine the cause of evolution, the object of research, the conditions of the model applicability.
10. For the mathematical model of the missile flight, determine the input parameters, causal relationships, state functions.
11. For the mathematical model of the glider flight, determine the object of study, coordinate system, output parameters.
12. For the mathematical model of the glider flight, determine the state functions, causal relationships, input parameters.
13. For the mathematical model of the glider flight, determine the reason for the evolution, input parameters, the independent variables.
14. For the mathematical model of the glider flight, determine the cause of evolution, the object of research, output parameters.
15. For the mathematical model of the glider flight, determine the input parameters, causal relationships, state functions.

**Task 2. Mechanical oscillations. Oscillations of pendulum and spring**

**Methodical instructions**

In Lecture 2, a mathematical model of the process of pendulum oscillations was considered. In particular, there is given the derivation of the equation for the oscillation of the pendulum, its solution is given, the law of conservation of the oscillation energy is derived, the equilibrium position of the system is established, mathematical models of the oscillation of the pendulum in the presence of friction, as well as under the action of an external force, are considered.

The first six variants of the task relate to the energy characteristics and the equilibrium position of the pendulum in the presence of friction. When performing these tasks, one should be based on the formulas given in the lecture for calculating the kinetic and potential energy and the concept of the equilibrium position of a pendulum in the absence of friction considered in the lecture.

The rest of the tasks are associated with a mathematical model of the spring oscillation process. The derivation of this mathematical model is given in the Appendix to Lecture 2. When completing the task, one should start from the analogy between the model of pendulum oscillation described in detail in the lecture and the model of spring oscillation considered in the task.

In all tasks, it is required not only to give the corresponding results, but also indicate their physical interpretation.

**Variants**

1. For the equation of the pendulum oscillation, establish the law of change in kinetic energy. Explain the obtained results.
2. Establish the law of potential energy change for the equation of the pendulum oscillation. Explain the obtained results.
3. Find equilibrium positions for a mathematical model of general (not small) pendulum oscillations. Explain the obtained results.
4. Find equilibrium positions for the mathematical model of the pendulum oscillations with friction. Explain the obtained results.
5. Find the law of change in kinetic energy for a mathematical model of the pendulum oscillations with friction. Explain the obtained results.
6. For a mathematical model of the pendulum oscillations with friction, find the law of change in kinetic energy. Explain the obtained results.
7. Find a solution to the equation of the spring oscillation, which is at the initial moment in equilibrium and has a nonzero velocity. Explain the obtained results.
8. Find a solution to the equation of the spring oscillation that is at the initial time in a state other than equilibrium and has zero velocity. Explain the obtained results.
9. For the equation of the spring oscillation, obtain the law of conservation of energy.
10. Give the equation of the oscillation spring in the presence of friction. Establish equilibrium positions for it. Explain the obtained results.
11. Find a solution to the equation of oscillation in the presence of friction with zero initial states. Explain the obtained results.
12. Give solutions to the equation of forced oscillation of a spring in the presence of friction. Explain the obtained results.
13. Give solutions to the equation of forced oscillation of the spring in the absence of friction. Explain the obtained results.
14. For a mathematical model of spring oscillation with friction, find the law of change in kinetic energy. Explain the obtained results.

**Task 3. Electrical oscillations. Oscillations of electrical circuit**

**Methodical instructions**

In Lecture 3, mathematical models of processes associated with an electrical circuit were considered. The main result here is the presence of a deep analogy between the mechanical processes associated with the movement of a pendulum and a spring, and the behavior of an electrical circuit. Mathematically, these processes are identical, being described by the same equations. When performing the next task below, one should be based on this analogy.

In fact, any task can be completed in three stages:

1. Translation of the delivered task from the electrical language to the mechanical one.

2. Using the already known corresponding result from the previous lecture.

3. Reverse translation of the result obtained from mechanical language into electrical one.

**Variants**

1. Establish the law of change in the electrical energy of a circuit with resistance, if at the initial moment of time the charge and current are equal to zero. Explain the results obtained.
2. Establish the law of change of the magnetic energy of the circuit with resistance, if at the initial moment of time the charge and current are equal to zero. Explain the results obtained.
3. Determine the law of change in the current in the circuit, with a discharged capacitor and some initial current.
4. Determine the law of charge change in the circuit, with a discharged capacitor and some initial current.
5. Establish the change over time of the magnetic energy of the circuit with resistance. Explain the results obtained.
6. Establish the change over time of the electrical energy of the circuit with resistance. Explain the results obtained.
7. Establish an equilibrium position for the charge in an electrical circuit with resistance. Explain the results obtained.
8. Establish an equilibrium position for the charge in the electrical circuit. Explain the results obtained.
9. Establish the change over time in the potential energy of the circuit. Explain the results obtained.
10. Establish the change over time in the electrical energy of the circuit. Explain the results obtained.
11. Establish the equilibrium position of the current in the electrical circuit. Explain the results obtained.
12. Establish the equilibrium position of the voltage in the electrical circuit. Explain the results obtained.
13. Establish the change over time in the current for the electric circuit with the given initial values ​​of the charge and current. Explain the results obtained.
14. Establish the change over time of the voltage for the electric circuit with the given initial values ​​of charge and current. Explain the results obtained.

**Task 4. Mathematical model of chemistry. Chemical kinetic problems**

**Methodical instructions**

In Lecture 4, mathematical models of chemical processes were considered, which characterize the change over time of the initial substances and products of chemical reactions. In the following tasks, it is required to write down a mathematical model for a specified system of chemical reactions, which is a system of differential equations for all substances participating in the reactions, with the corresponding initial conditions. As a sample, here is a mathematical model for the Lotka reaction system. Indicate the order of each of the considered reactions. In all tasks, each of the reactions is characterized by its own reaction velocity *ki*, where *i* is the number of the reaction from the task.

**Variants**

1. A + 2B → 2C + D, C + 2D → A, 3A + C → 2B.
2. 2A + B → A + D, C + 2A → B, 3B + D → 2A.
3. 3A + D → 2B + C, 2B + D → 2A + 3C, A + 2C → 3B.
4. A + 3B → 3C + D, B + 2C → D, 2B + D → A.
5. 2A + C → 2B + 3D, C + 2B → 2A, 3A → 2B + C.
6. 2A + B → A + D, 2D + A → C, 3B + D → 2C.
7. A + 2D → C, 3B + A → 2A, D + 2C → 3B.
8. A + B + C → D, B + 2C → 3D, B + D → A.
9. 2A + B → D, 3C + D → A + B, A + 2C → 2B + D.
10. A + C → 2D, 2C + A → 2B, 3D + B → 2A.
11. A + 2D → 2C, 2C + B → D, D + 3C → 2B.
12. 2A + D → 2C + B, 2B + C → D, 2C + D → A.
13. A + C + 2D → 2B, 2C + B → 3A, 3D → 2B + C.
14. A + 2B → C, D + A → 2C, 3B + C → 2A.

**Task 5. Mathematical model of chemistry. Symbiosis model**

**Methodical instructions**

In Lecture 5, mathematical models of biological processes that characterize the change over time in the number of biological species under different conditions of their existence were considered. In the following tasks, it is required for the symbiosis model to select specific numerical values of all parameters of the system at which the specified effect is realized and to explain the results obtained from the point of view of biology. In a number of variants, the described situation is impossible. In this case, the reason for the impossibility of the situation should be explained.

**Variants**

1. Choose the values ​​of the parameters at which both state functions increase monotonically, if this is possible. Give an interpretation of the results obtained or explain why this is impossible.

2. Choose the values ​​of the parameters at which both state functions decrease monotonically, if this is possible. Give an interpretation of the results obtained or explain why this is impossible.

3. Select the values ​​of the parameters at which the first state function first increases and then decreases, and the second decreases monotonically, if this is possible. Give an interpretation of the results obtained or explain why this is impossible.

4. Choose the parameter values ​​at which the first state function first decreases and then increases, and the second decreases monotonically, if this is possible. Give an interpretation of the results obtained or explain why this is impossible.

5. Choose the parameter values ​​at which the second state function first increases and then decreases, and the first decreases monotonically, if this is possible. Give an interpretation of the results obtained or explain why this is impossible.

6. Choose the parameter values ​​at which the second state function first decreases and then increases, and the first decreases monotonically, if this is possible. Give an interpretation of the results obtained or explain why this is impossible.

7. Choose the values ​​of the parameters at which the first state function decreases monotonically, and the second monotonically increases, if this is possible. Give an interpretation of the results obtained or explain why this is impossible.

8. Choose the parameter values ​​at which the first state function first increases and then decreases, and the second increases monotonically, if this is possible. Give an interpretation of the results obtained or explain why this is impossible.

9. Choose the values ​​of the parameters at which both state functions monotonically tend to the equilibrium position, if this is possible. Give an interpretation of the results obtained or explain why this is impossible.

10. Select the values ​​of the parameters at which the first state function first decreases and then increases, and the second monotonically increases, if this is possible. Give an interpretation of the results obtained or explain why this is impossible.

11. Choose the values ​​of the parameters at which the second state function first decreases and then increases, and the first monotonically increases, if this is possible. Give an interpretation of the results obtained or explain why this is impossible.

12. Choose the parameter values ​​at which the second state function first decreases and then increases, and the first increases and then decreases, if this is possible. Give an interpretation of the results obtained or explain why this is impossible.

13. Choose the parameter values ​​at which the second state function first decreases and then increases, and the first increases monotonically, if this is possible. Give an interpretation of the results obtained or explain why this is impossible.

14. Select the values ​​of the parameters at which the state of the system does not change at all, if this is possible. Give an interpretation of the results obtained or explain why this is impossible.

**Task 6. Mathematical model of economics. Competition model**

**Methodical instructions**

In Lecture 6, mathematical models of economic processes were considered. The purpose of this assignment is to study a **competition model** that also has biological meaning. This assumes a description of the evolution of the system for one of its possible variants, presented in **Figure 8.1** of the corresponding lecture or on **Slide 16** of the presentation, which shows the phase curves in the competition model. The tasks indicate the options for combining the coefficients of the equation (a, b,c) and the number of the phase curve corresponding to this option and determined by the initial states of the system (1,2,3,4). It is required to explain the meaning of this combination of parameters and to describe the development of events from the initial state of the system to the end of the process. All explanations should be carried out in both **economic** and **biological** interpretation.

**Variants**

1. Variant *а*, curve 1.
2. Variant *а*, curve 2.
3. Variant *а*, curve 3.
4. Variant *а*, curve 4.
5. Variant *b*, curve 1.
6. Variant *b*, curve 2.
7. Variant *b*, curve 3.
8. Variant *b*, curve 4.
9. Variant *с*, curves 1.
10. Variant *c*, curves 2.
11. Variant *c*, curve (line) 3.

**Task 7. Mathematical model of social sciences. Niche model**

**Methodical instructions**

Lecture 7 analyzed mathematical models in the social sciences. The purpose of this assignment is to study a niche model that also has biological and economic meaning. In the task, it is required to select specific parameters of the system (equation coefficients and initial states) in the variables *u*, *v* so that the effect described in the task is observed. Describe the corresponding evolution of the system in the interpretation specified in the assignment.

**Variants**

1. Both state functions increase monotonically and reach a nonzero equilibrium position (political science)

2. The first state function decreases to zero, and the second first decreases and then increases (economy)

3. The second function of the state decreases to zero, and the first decreases and then increases (biology)

4. Both state functions decrease monotonically and reach a nonzero equilibrium position (economics)

5. The first function of state increases monotonically, and the second first increases and then decreases to zero (political science)

6. The second function of the state monotonically increases, and the first increases and then decreases to zero (biology)

7. The first state function monotonously increases, and the second monotonically decreases to zero (economy)

8. The first state function decreases monotonically to zero, and the second increases monotonically (political science)

9. The first function of the state increases monotonically, and the second decreases monotonically to a nonzero equilibrium position (biology)

10. The first state function decreases monotonically to a nonzero equilibrium position, and the second monotonically increases (economics)

11. Both state functions increase monotonically and reach a non-zero equilibrium position (biology)

12. The first function of the state first decreases and then increases, and the second decreases to zero (political science)

13. The first function of the state decreases to zero, and the second first decreases and then increases (economy)

14. The first function of state increases monotonically, and the second decreases monotonically to zero (biology)

**Task 8. Heat equation**

**Methodical instructions**

In Lecture 8, mathematical models of transfer processes were considered, which are the first and second boundary value problems for a homogeneous and inhomogeneous heat equation. In the following tasks, specific conditions for the flow of the heat process in a thin homogeneous body of a given length are described

The following steps are required:

1. Write a mathematical model of the process.

2. Using ready-made formulas for solutions from the text of a lecture or presentation, give a solution to the set boundary value problem.

3. Make sure that this is indeed a solution to the problem by substituting it into the equation and boundary conditions.

4. Indicate how and why there is a change in body temperature with time at its various points, taking as a sample the corresponding descriptions from the lecture.

**Variants**

1. A body of unit length in the absence of external heat sources. The thermal diffusivity is 1. At the ends, the body is thermally insulated. At the initial moment of time, the temperature is distributed according to the cosπx law.

2. A body of unit length in the absence of external heat sources. The coefficient of thermal diffusivity is 4. Zero temperature is maintained at the ends of the body. At the initial moment of time, the temperature is distributed according to the 2sinπx law.

3. A body of unit length in the presence of an external heat source distributed according to the cosπx law. The thermal diffusivity is 1. At the ends, the body is thermally insulated. At the initial moment of time, the temperature is everywhere equal to zero.

4. A body of unit length in the presence of an external heat source distributed according to the sinπx law. The coefficient of thermal diffusivity is 4. Zero temperature is maintained at the ends of the body. At the initial moment of time, the temperature is everywhere equal to zero.

5. Body of length π in the absence of external heat sources. The thermal diffusivity is 4. At the ends, the body is thermally insulated. At the initial moment of time, the temperature is distributed according to the cos*x* law.

6. Body length 2π in the absence of external heat sources. The coefficient of thermal diffusivity is 1. Zero temperature is maintained at the ends of the body. At the initial moment of time, the temperature is distributed according to the sin(x/2) law.

7. A body of length 2π in the presence of an external heat source distributed according to the   
cos(x/2) law. The thermal diffusivity is 1. At the ends, the body is thermally insulated. At the initial moment of time, the temperature is everywhere equal to zero.

8. Body length 2π in the absence of external heat sources. The thermal diffusivity is 2. At the ends, the body is thermally insulated. At the initial moment of time, the temperature is distributed according to the law cos(x/2).

9. Body length 2π in the absence of external heat sources. The coefficient of thermal diffusivity is 2. Zero temperature is maintained at the ends of the body. At the initial moment of time, the temperature is distributed according to the sin(x/2) law.

10. Body of length π/2 in the absence of external heat sources. The thermal diffusivity is 1. At the ends, the body is thermally insulated. At the initial moment of time, the temperature is distributed according to the cos2x law.

11. Body of length π/2 in the absence of external heat sources. The coefficient of thermal diffusivity is 1. Zero temperature is maintained at the ends of the body. At the initial moment of time, the temperature is distributed according to the sin2x law.

12. A body of length π/2 in the presence of an external heat source distributed according to the cos2x law. The thermal diffusivity is 1. At the ends, the body is thermally insulated. At the initial moment of time, the temperature is everywhere equal to zero.

13. A body of length π/2 in the presence of an external heat source distributed according to the sin2x law. The coefficient of thermal diffusivity is 1. Zero temperature is maintained at the ends of the body. At the initial moment of time, the temperature is everywhere equal to zero.

14. Body of unit length in the absence of external heat sources. The thermal diffusivity is 3. Zero temperature is maintained at the ends of the body. At the initial moment of time, the temperature is distributed according to the 2sinπx law.

**Task 9. Transfer processes**

**Methodical instructions**

In Lecture 9, mathematical models of various transfer processes were discussed. The following tasks describe the specific conditions for a complex transfer process, when, on the one hand, there is a certain chemical, biological or economic process considered before in the first part of the course, and, on the other hand, events occur in a certain one-dimensional area, as a result of which the corresponding process is implemented transfer. It is required to give a complete mathematical model of the process, including a system of state equations with appropriate initial and boundary conditions. As a model, you can use the models of the chemical reaction in a certain area and the migration of competing biological species discussed in the lecture.

**Variants**

1. Given the chemical reaction A + 2B→C. The initial concentrations of all substances are known. At the left end, the concentration areas are known. The right end is isolated.

2. The population of predators and prey migrating over a certain territory is considered. The initial number of species is known. The area is isolated at both ends.

3. Consider two competing firms that distribute the same product over a certain territory. The initial production volume of both firms is known. The area is isolated at both ends.

4. The chemical reaction А→В+С is given. The initial concentrations of all substances are known. At the right end of the area, all concentrations are known. The left end is isolated.

5. Consider a population of two species in symbiosis, migrating over a certain territory. The initial number of species is known. The abundance of both species at the boundary is known.

6. Consider two cooperating firms that distribute goods over a certain territory. The initial production volume of both firms is known. The area is isolated at both ends.

7. A chemical reaction 2A+B→C is given. The initial concentrations of all substances are known. The area is isolated at both ends.

8. Consider a competing population migrating over a certain territory. The initial number of species is known. The first species has a given number at the left end of the region and cannot go beyond the right border, and the second, vice versa.

9. Consider two competing firms that distribute the same product over a certain territory. The volume of products manufactured by both firms at the initial moment of time and at the border of the considered area are known.

10. Chemical reactions А→2В, В→С are given. The initial concentrations of all substances are known. The area is isolated at both ends.

11. The population of predators and prey migrating over a certain territory is considered. The abundance of both species at the initial moment of time and at the border of the region is known.

12. Two cooperating firms are considered, distributing goods over a certain territory. The initial and boundary values ​​of the volume of products manufactured by both firms are known.

13. A chemical reaction A+B→C is given. The initial concentrations of all substances are known. At the left end, the flow of substance A is known, as well as the concentrations of substances B and C. The right end is isolated.

14. The population of predators and prey migrating over a certain territory is considered. The initial number of species is known. The predator has a given number at the left end of the area and cannot go beyond the right border, while the prey does the opposite.

**Task 10. Wave processes**

**Methodical instructions**

In Lecture 10, mathematical models of string vibrations were considered, which are the first (fixing the ends of the string) and the second (free ends of the string) boundary value problems for the corresponding homogeneous equation.

The following steps are required:

1. Write down a mathematical model of the process.

2. Using ready-made formulas for solutions from the text of a lecture or presentation, give a solution to the boundary value problem.

3. Make sure that this is indeed a solution to the problem by substituting it into the equation and boundary conditions.

4. Explain how and why vibrations of a string occur with time at its various points, taking as a model the corresponding descriptions from the lecture. Consider the position an velocity of different points of string.

**Variants**

1. String of unit length. Coefficient a = 4. The string is fixed at the ends. At the initial moment of time, the shape of the string is sinπx. The initial velocity is zero.
2. String of unit length. Coefficient a = 1. The ends of the string are free. At the initial moment of time, the shape of the string is cosπx. The initial velocity is zero.
3. String of length π. Coefficient a = 1. The string is fixed at the ends. At the initial moment of time, the string is in equilibrium. The initial velocity is distributed according to the sinx law.
4. String of unit length. Coefficient a = 2. The ends of the string are free. At the initial moment of time, the string is in equilibrium. The initial speed is distributed according to the cosπx law.
5. String of length π. Coefficient a = 3. The string is fixed at the ends. Initially, the shape of the string is -sinx. The initial velocity is zero.
6. String of unit length. Coefficient a = 2. The ends of the string are free. At the initial moment of time, the shape of the string is: -cosπx. The initial velocity is zero.
7. String of length 1. Coefficient a = 4. The string is fixed at the ends. At the initial moment of time, the string is in equilibrium. The initial velocity is distributed according to the law: -sinπx.
8. String of length 1. Coefficient a = 2. The string is fixed at the ends. At the initial moment of time, the string is in equilibrium. The initial velocity is distributed according to the law: -sinπx.
9. String of length π. Coefficient a = 3. The ends of the string are free. At the initial moment of time, the shape of the string is cos2x. The initial velocity is zero.
10. String of length π. Coefficient a = 2. The string is fixed at the ends. At the initial moment of time, the shape of the string is sin2x. The initial velocity is zero.
11. String of length 1. Coefficient a = 2. The ends of the string are free. At the initial moment of time, the string is in equilibrium. The initial velocity is distributed according to the law: -cos2πx
12. String of length 1. Coefficient a = 3. The string is fixed at the ends. At the initial moment of time, the string is in equilibrium. The initial velocity is distributed according to the law: sin2πx
13. String of length π. Coefficient a = 2. The ends of the string are free. The initial velocity is zero. At the initial time, the shape of the string is -cos2x.
14. String of unit length. Coefficient a = 1. The ends of the string are free. At the initial moment of time, the shape of the string is 2cos2πx. The initial velocity is zero.

**Task 11. Field theory**

**Methodical instructions**

In Lecture 11, mathematical models of electrostatic and gravitational fields were considered. In this case, the potentials of these fields are described by the Poisson equation. It is known that the electrostatic field in the absence of charges and the gravitational field in vacuum are described by the Laplace equation.

In the case of a point source of a gravitational or electrostatic field, due to spherical symmetry, the Laplace equation is reduced to an ordinary differential equation. Its solution at an arbitrary point is determined by the distance from this point to the field source. For a homogeneous wire, due to cylindrical symmetry, the Laplace equation is also reduced to an ordinary differential equation. Its solution at an arbitrary point is determined by the distance from that point to the wire.

In the following tasks, an electrostatic or gravitational field in three-dimensional space is considered in the case of spherical or cylindrical symmetry. The point where the point source is located, or the straight line corresponding to the direction of the wire, is indicated. The charge (for the wire - the charge density) of the source of the gravitational field or the mass of the source of the gravitational field is known.

The following steps are required:

1. Write down the equation for the field potential with the indicated type of symmetry.

2. Using ready-made formulas for solutions from the text of a lecture or presentation, give a solution to the set boundary value problem.

3. Make the change of variables by placing a point source at the origin or by pointing the wire along the *z* axis.

4. Find the value of the potential of the corresponding field at the point specified in the task.

5. Comment on the results.

**Variants**

1. Consider the electrostatic field of the charge *e* = 2, located at a point with coordinates (1,1,1). Find the value of the field potential at the point (1,2,3).

2. Consider the gravitational field of mass *m* = 2, located at a point with coordinates (2,1,2). Find the value of the field potential at the point (1,0,3).

3. The electrostatic field of a wire with a charge density *e* = 3 passing through the point (2,0,0) parallel to the *z* axis is considered. Find the value of the field potential at the point (0,2,0).

4. Consider the electrostatic field of the charge *e* = 1, located at a point with coordinates (1,0,1). Find the value of the field potential at the point (1,2,2).

5. Consider the gravitational field of mass *m* = 3, located at a point with coordinates (1,2,2). Find the value of the field potential at the point (1,0,3).

6. The electrostatic field of a wire with a charge density *e* = 2 passing through a point (0,1,0) parallel to the *z* axis is considered. Find the value of the field potential at the point (1,2,0).

7. Consider the electrostatic field of the charge *e* = 3, located at a point with coordinates (1,1,0). Find the value of the field potential at the point (1,2,2).

8. Consider the gravitational field of mass *m* = 3, located at a point with coordinates (2,1,1). Find the value of the field potential at the point (1,1,3).

9. The electrostatic field of a wire with a charge density *e* = 2 passing through the point (1,1,0) parallel to the *z* axis is considered. Find the value of the field potential at the point (1,2,0).

10. Consider the electrostatic field of the charge *e* = 1, located at a point with coordinates (0,1,1). Find the value of the field potential at the point (1,2,1).

11. We consider the gravitational field of mass m = 3 located at the point with coordinates (2,1,0). Find the value of the field potential at the point (1,0,2).

12. The electrostatic field of a wire with a charge density *e* = 1 passing through the point (1,2,0) parallel to the *z* axis is considered. Find the value of the field potential at the point (0,1,1).

13. The electrostatic field of the charge *e* = 4, located at the point with coordinates (0,0,1), is considered. Find the value of the field potential at the point (1,1,1).

14. The gravitational field of mass *m* = 2, located at a point with coordinates (1,0,1), is considered. Find the value of the field potential at the point (1.0, -1).

**Mathematical modelling**

**Homework**

**Task 12. Variational principles**

**Methodical instructions**

Lecture 12 examined various issues related to the application of variational principles. Using the material of the lecture and presentation, you need to complete the following tasks.

**Variants**

1. Write down the first integral in the problem of a curve of minimum length.

2. A spring is considered, on which the elastic force acts according to the law *F=-kx*, where *k* is the coefficient of elasticity, and *x* is the deviation from the equilibrium position. Using the principle of least action and the Euler equation, obtain the equation of motion.

3. For the motion of a body on a plane under the action of a constant force, obtain the law of conservation of energy using the first integral of the system.

4. The motion of a body of variable mass on a plane under the action of some force is considered. Using the principle of least action and the Euler equation, obtain the equations of motion.

5. Write down the first integral in the brachistochrone problem.

6. Using Fermat's principle, establish the trajectory of light on a plane in a homogeneous medium from one arbitrary point to another.

7. A ray of light moves on a plane from a point with coordinates (-5.5) at an angle of 300 to the   
*x*-axis. It is known that the speed of light in the lower half-plane is half that in the upper one. Using Fermat's principle, establish what the vertical coordinate of the point where the light with the horizontal coordinate *x*=5 will hit.

8. Rectilinear motion of a body of variable mass under the action of a known variable force is considered. Using the principle of least action, obtain the equation of motion. In the case of constant force and mass, obtain the law of conservation of energy using the first integral of the system.

9. Write down the Euler equation for the brachistochrone problem.

10. The motion of a body of constant mass on a plane under the action of a constant force is considered. Using the first integral of the system, obtain the energy conservation law.

11. A spring is considered, on which the elastic force acts according to the law *F=-kx*, where *k* is the coefficient of elasticity, and *x* is the deviation from the equilibrium position. Using the first integral of the system, obtain the energy conservation law.

**Task 13. System identification**

**Methodical instructions**

In Lecture 14, the concept of systems identification and methods for solving identification problems were considered. A description of the system is given below for each variant. Follow these steps.

1. Give the formulation of the direct problem with an indication of the unknown quantities included in it.

2. Give a complete statement of the corresponding inverse problem.

3. Reduce the inverse problem to the corresponding optimization problem by determining the minimized functional with an indication of its arguments.

**Variants**

1. The process of diffusion in a given one-dimensional region is considered. The initial concentration of the substance is known. At the left end of the body, the concentration and diffusion flux are set. There is no information on the right end. Additionally, the concentration is known at three fixed points over a certain time interval.

2. The process of string vibration is considered. The initial position of the string is known, but the initial velocity is not. The left end of the string moves according to a given law. The law of movement of the right end of the string is unknown. Additionally, the position of the string at a fixed time is known.

3. The process of heat transfer in a given one-dimensional region is considered. At the left end, the heat flux is known, and at the right, the law of temperature change. The initial body temperature is known. The unknown is the thermal conductivity coefficient. Additionally, the law of temperature change at some internal point is known.

4. The process of forced oscillations of a pendulum in the presence of friction is considered. The initial position and velocity of the pendulum are known. The coefficient of friction is unknown. Additionally, the position of the pendulum is known at three fixed times.

5. The process of goods distribution over a given one-dimensional area is considered. At the left end, the flux of goods is known, and at the right, the law of change in the density of goods. The initial density of the goods is known. The unknown is the coefficient of transfer of goods. Additionally, we know the distribution of the density of the goods at the final time.

6. The process of diffusion in a given one-dimensional region is considered. The initial concentration of the substance is known. At the right end of the body, the concentration and diffusion flux are known. There is no information on the left end. In addition, the concentration is known at two fixed points at a certain time interval.

7. The process of charge transfer in a given one-dimensional region is considered. At the left end, the flux of charges is known, and at the right, the law of change in the charge density. The initial charge density is known. The unknown is the coefficient of electrical conductivity. In addition, the distribution of the charge density at the final time is known.

8. The process of migration of a biological species in a given one-dimensional area is considered. The left area is isolated, and on the right, the law of change of the species density of the view is given. The initial value of the species density is known. The unknown is the transfer coefficient of the species. Additionally, we know the law of change of the species density of the form at some interior point.

9. The process of string vibration is considered. The initial position of the string is unknown, but the initial velocity is given. The left end of the string moves according to a given law, and the right end of the string is free. String tension is unknown. Additionally, the position of the string at the final time is known.

10. The process of migration of a biological species in a given one-dimensional area is considered. The right end of the region is isolated, and on the left, the law of change of the species density is known. The initial density of the species is unknown. Additionally, the law of variation of the density of the species at three interior points.

11. The process of charge transfer in a given one-dimensional region is considered. At the right end, the flux of charges is known, and the left end is isolated. The initial charge density is unknown. Additionally, the change in the charge density at two fixed internal points is known.